

Form an integral equation corresponding to the differential equation given by

$$\frac{d^3y}{dx^3} + x \frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1$$

with initial conditions $y(0) = 1 = y'(0)$,

$$y''(0) = 0$$

Soln

The given differential equation is

$$\frac{d^3y}{dx^3} + x \frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1 \quad \text{--- (1)}$$

with initial conditions

$$y(0) = 1 = y'(0), \quad y''(0) = 0 \quad \text{--- (2)}$$

Let us suppose

$$\frac{d^3y}{dx^3} = u(x) \quad \text{--- (3)}$$

Integrating (3) b.s.w.r. to x from 0 to x
we get

$$\frac{d^2y}{dx^2} = \int_0^x u(t) dt \quad \text{--- (4)}$$

Again integrating w.r. to x from 0 to x
we get

$$\frac{dy}{dx} = \int_0^x (x-t) u(t) dt + 1$$

Similarly, we get

$$y = \frac{1}{2} \int_0^x (x-t)^2 u(t) dt + x + 1 \quad \text{--- (5)}$$